A comparative study of full factorial and central composite designs through the machining of aluminium alloy

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Abstract. The primary task of the design of experiments (DOE) is to minimise the number of the experimental setups in order to minimise the working time and costs, and maximise the information about the analysed process. Different experiment designs exist, which require different experimental setups, thus those has an influence on the analysed responses. The primary objective of the present study was a comparative analysis of different design of experiments. A full factorial (FF) and a central composite inscribed (CCI) design were used in order to analyse the machinability of a 6082 aluminium alloy. The cutting speed and the feed rate were chosen as factors in order to analyse their influence on the cutting force. The face milling experiments were conducted on a Kondia B640 machining centre. The cutting force was measured with a KISTLER 9257BA load cell with 8 kHz sampling frequency. The data were collected with the KISTLER Dyno Ware software. The results were analysed with the Microsoft Excel and the Minitab 17 software. Second-degree mathematical models were used to describe the responses by the means of the response surface methodology (RSM). The main effects and the interactions of the factors were analysed with the analysis of variance (ANOVA). The collected information, experiment cost and time were compared and discussed in order to realise the differences between the full factorial and a fractional factorial design.

Keywords: Design of Experiments; ANOVA, Central Composite Inscribed; Machinability, Milling, Cutting force

1 Introduction

Today the efficiency, cost reduction, and a reduced time to market became essential requirements, which are justified by customer needs and environmental awareness. The realisation of these needs can be done in a lot of different ways during the design process, among which the structure- and process optimization methods can be highlighted. Different statistical methods of DOE (Design of Experiments) have been studied in this paper. The primary objective of DOE is to determine what kind of experimental settings are needed for our experiments to gather the most information...
about the studied process with the minimal effort. The Fig. 1. represents the structure of the DOE, which is the object of the research.

\[ y = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{\frac{n(n-1)}{2}} b_{ij} x_i x_j + \delta \]  

(1)

In the Eq. (1) \( x_i \) are factors, \( x_i x_j \) are interaction of those factors, \( b_0 \) is a constant, \( b_i \), \( b_{ii} \), \( b_{ij} \) are regression coefficients, \( \delta \) is an accidental error, and \( y \) is desired optimisation parameter. The main effects and interactions have been analysed by the Analysis of Variance (ANOVA) statistical method. The obtained information content, and the time and cost expenditures have been compared, and the consequences of the experiment were discussed.

2 Experimental settings

For the measurement of the cutting force components an aluminium alloy type 6028 with geometry of 60x60x100 mm has been tested by chamfering. The experiments were carried out on a three-axis B640 type Kondia machining centre with NCT100 control, where a MITSUBISHI type AQXR212SA20S ø21 mm double-edged end mill was
used. The cutting force has been detected with a KISTLER type 9257 three-component load cell at a sampling frequency of 8 kHz, while the data was collected by the KISTLER Dyno Ware software. The cooling of the workpiece/fixture/machine tool/cutting tool (WFMT) system has been ensured by MQL lubrication, where a TRIM type ML26 lubricant was applied. The used machine tool and the machining environment are shown in Fig. 2.

![Machine tool and environment](image)

**Fig. 2.** The used machine tool and the machining environment

Two experiment design methods, the full factorial (FF) and the central composite inscribed (CCI) methods have been examined, while the experiment plans were made in the Minitab 17 statistical software. The cutting speed \(v_c\) and the feed rate \(v_f\) were chosen for factors. Based on the literature research [7-11], these are the factors which have the biggest impact on the machinability. When selecting the interpretation range of the factors technological and economical considerations (machine tool, tool geometry, grip stability) and economic limitations were also taken into account. The experimental design factor levels and the interpretation range of the variable factors are presented in the Table 1.

<table>
<thead>
<tr>
<th>Factor levels of the variable factors</th>
<th>Table 1</th>
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<tbody>
<tr>
<td><strong>FF</strong></td>
<td>(v_c) [m/min]</td>
</tr>
<tr>
<td></td>
<td>(v_f) [mm/rot.]</td>
</tr>
<tr>
<td><strong>CCI</strong></td>
<td>(v_c) [m/min]</td>
</tr>
<tr>
<td></td>
<td>(v_f) [mm/rot.]</td>
</tr>
</tbody>
</table>

During the experiments the presence of other factors has also had to be taken into account. Values of these factors have been kept at the same level so that their effect on the optimisation parameters did not change. That way the axial \(a_p=3\ mm\) and the
radial \((a_c=5 \text{ mm})\) depth of cut (DOC) were fixed during the experiments, and only climb milling was applied.

### 3 Results

The evaluation of the results was performed in Minitab 17 and Microsoft Excel software packages. On the obtained high frequency signal Gaussian frequency filtering was performed, where the actual point was calculated by its own surrounding’s weighted average. A \(t_s=1 \text{ second}\) section of the filtered signal was evaluated, so the force components \((F_{x,\text{max}}; F_{y,\text{max}})\) and the resultant force \((F_{\text{max}})\) was also calculated using the data of that particular section. In the following part of the study these values were used to draw the graphs and to analyse the statistical results also. The statistical analysis has been performed at a 95% significance level.

#### 3.1 Full Factorial (FF)

The FF results can be seen in the Fig. 4-5. The response surfaces presented in the Fig. 5. are made up of planes with different slopes. The boundaries for these planes are provided by the set factor levels. On the graph the optimisation parameters \((F_{x,\text{max}}; F_{y,\text{max}}, F_{\text{max}})\) are illustrated as functions of the variable factors \((v_c, v_f)\).

![Fig. 4. The maximal cutting force and its components as a function of the variable factors in the full factorial experiment design during chamfering.](image_url)
As in the Figures can be seen, increasing the value of the feed rate increases the strength values, but the effect of the cutting speed is less prominent. In the observed factor space the highest values have been obtained at a high feed rate and a low cutting speed. Visual results of the ANOVA experiment are shown in the Fig. 5.

![Fig. 5. Main-impact and interaction plots used the full factorial experiment design](image)

From the main impact study of the variable factors (Fig. 5, left graph) it can be seen that the feed rate has a great effect on the response, while the effect of the cutting speed is low. At the interactions (Fig. 5, right graph) it can be observed that the lines on the graph are nearly parallel, so the cutting speed and the feed rate has a low interaction.

### 3.2 Central Composite Inscribed (CCI)

The response surfaces obtained by the RSM method (Fig. 6.), are similar to the full factorial experiment results. In the function of the feed rate the cutting force has increased significantly, and it can be observed that by increasing the cutting speed the response has only slightly decreased. Fig. 7 shows the main and the cross effects. The results are similar to the results obtained by the Full Factorial method: At the main effect study it can be concluded that the feed rate has a significant effect on the optimisation parameter, while the effect of the cutting speed is low. From the graph of the latter it can be seen that it has a decreasing tendency, but with the increase of its values the monotonicity becomes uncertain. When examining interaction between the variable factors, it can be seen that they have just little effect on each other.
3.3 Comparison

The results from the two experiment design methods were compared. The maximum and the minimum locations and values of the optimisation parameter, and the cutting time needs with its variations were also determined. These are summarised in Table 2.
Max. and min. locations, values, difference between the two designs, and the time spent

Table 2.

<table>
<thead>
<tr>
<th></th>
<th>Full Factorial (FF)</th>
<th>Central Composite Inscribed (CCI)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F_{x, \text{max}}$</td>
<td>$F_{y, \text{max}}$</td>
<td>$F_{\text{max}}$</td>
</tr>
<tr>
<td>Max value</td>
<td>316.44</td>
<td>485.17</td>
<td>537.33</td>
</tr>
<tr>
<td>Max location $v_c$</td>
<td>320</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Max location $v_f$</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Min value</td>
<td>66.44</td>
<td>168.90</td>
<td>171.61</td>
</tr>
<tr>
<td>Min location $v_c$</td>
<td>298</td>
<td>298</td>
<td>298</td>
</tr>
<tr>
<td>Min location $v_f$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Exp. Time</td>
<td>11 min</td>
<td></td>
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The differences between the experiment design methods’ results show that the greater difference has appeared in the location of the cutting speed. This may be related to the main and interaction studies, since the factor has no significant effect on the responses, so its determination involves greater uncertainty. However between the two experiment design methods exists a significant difference when considering the time spent. In the Table 2 the given times only represent the cutting time, but it is understood that with a higher experiment count setting the auxiliary times will also be longer.

Fig. 8. Deviation of the CCI method’s results compared to the FF method in the function of variable factors
Fig. 8 illustrates the deviation of the CCI method’s results compared to the FF method. The deviation of the CCI experiment design’s error shows that higher-value setting parameters increase the deviation, so the estimation uncertainty of the results’ has grown. The deviation fluctuations were mostly present at high cutting speed values. By increasing the feed rate the range of variation remained nearly constant, while at higher cutting speeds it has nearly tripled towards higher values.

4 Summary

Two experiment design methods, the Full Factorial (FF) and the Central Composite Inscribed (CCI) were compared through machinability experimentations. For the variable factors the cutting speed and the feed rate were selected, and their effect on the cutting force as the optimisation parameter was investigated. The aim of our research was to carry out experiments using experiment design methods, and then compare their information content. The following conclusions can be drawn:

- At both experiment designs, the greatest cutting force has been obtained by the lowest cutting speed and highest feed rate values.
- The effect of the cutting speed on the optimisation parameter is not significant, while the feed rate has a significant effect; however there is no interaction between the variable factors. These statements are true for both experiment designs.
- For both experiment designs it can be concluded that the measurement uncertainty increases as the cutting speed increases, while it remains nearly constant with increasing feed rates.
- The results of the CCI experiment design show a maximum 10% difference over the entire factor space and a 70% less time spent when compared to the FF results. So the CCI method is more cost-effective, since it requires less experimental settings, and less time.

The aim of our further research is to compare other experiment design methods, analysing their time and cost-effectiveness, to assist the machinability research-development engineers in choosing the appropriate experimental designs.
References


